647510 **MSC1MatC104x** **Seat No :________**

M.Sc.(Maths.) Semester - 1 (*CBCS*) Examination
Oct/Nov. -2019 - [NEW COURSE]
Theory of Ordinary Differential Equations(Core (New))

Time: 2:30 Hours **Marks: 70**

Instructions:

1. All questions are compulsory.
2. Figures to the right indicate marks.

Q1) Answer any seven : **(7 x 2=14)**

1) Define Eigen Values and Eigen Vectors.
2) Find general solution of $y^4 + 16y = 0$ on $\mathbb{R}$.
3) Define Heavy Side Function and Show that Laplace Transform is linear.
4) Prove that e^{3t} and e^{-3t} are two Linearly Independent solution of $y'' - 9y = 0$ on $\mathbb{R}$.
5) State the condition of the solution of an Initial value Problem of a system of 1^{st} order linear differential equation.
6) If y_1, y_2 are solutions of $(1 - x^2)y'' - 2xy' + p(p-1)y = 0$ with the initial conditions $y_1(0) = 0, y_1'(0) = -1, y_2(0) = 1, y_2'(0) = 0$ then find $w(y_1, y_2)(\frac{1}{2})$.
7) State First Shifting Theorem and find $L(e^{at})$(z).
8) Determine the largest interval of exisistance of the solution for the IVP:
$$(t^2)y'' + ty' + (t^2 - 4)y = 0 \text{ with } y(-1) = 0; y'(-1) = 1$$
9) Let A and B be $n * n$ matrix and $AB = BA$ then $exp(A + B) = exp(A) * exp(B)$.
10) Define: Power Series and Irregular Singular Point.

Q2) Answer any two : **(2 x 7=14)**

1) State and prove Variation of constant formulae for scalar linear second order homogenous differential equation.
2) Prove that Eigen vectors corresponding to the distinct Eigen values of $n * n$ matrix are linearly independent in $\mathbb{R}^n$or $\mathbb{C}^n$.
3) Show that a) Gamma $(\frac{1}{2}) = \sqrt{\pi}$.
   b) Gamma $(n) = (n-1)!$, for every $n = 1,2,3, \dots \dots .., n-1$.

Q3) Both are compulsory : **(2 x 7=14)**

1) State and prove Gronwell's Inequality
2) Find the solution of the initial value problem$y' = ty$ with $y(0) = 1$ and$y'(0) = 0$.

OR

Q3) Both are Compulsory : **(2 x 7=14)**

1) Find Fundamental Matrix of $y' = A(t)y$ on $(-\infty, \infty)$ where $A(t) = \begin{bmatrix} 1 & 2 \\ 4 & 3 \end{bmatrix} \forall t \in (-\infty, \infty)$and find exp(tA) ; $\forall t \in (-\infty, \infty)$.
2) Find the particular solution of$y'' + y = sint$ on$\mathbb{R}$; $y(0) = 0$ and $y'(0) = 0$.

Q4) Answer any two : **(2 x 7=14)**

1) Justify weather the Legendre's equation$(1-t^2)y''-2ty'+n(n+1)y = 0$; (where n is constant) has a solution or not.
2) Solve the IVP: $y_1{}'=y_1+y_2+f(t)$and $y_2{}'=y_1+y_2$ with $y_1(t_0)=0=y_2(t_0)$ where $f: I \to \mathbb{R}$ is a continuous and $t \in I$.
3) a) Classify and locate all the singularities of
$$t^4(1-t^2)^3y'''+5t^5(1+t)y''-2t^2(1-t^2)y'+y=0$$
b) Prove that if $\emptyset$ is a solution of the I.V.P: $y'=f(t,y); y(t_0)=y_0$ if and only if $\emptyset$ is a solution of the Voltera's equation $y(t)=y_0\int_{t_0}^{t} f(s,y(s))ds$.

Q5) Answer any two : **(2 x 7=14)**

1) Find $L(t^n e^{ct})(z)$.
2) State and prove Laplace transform of integral.
3) Solve $L^{-1}\left(\frac{1}{z(z^2+4)^2}\right)$ using convolution theorem.
4) Solve:$y''+4y=3cost\ with\ y=1\ and\ y'=-2\ when\ t=0$.
5)

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